Guidelines for Estimation and Expression of Uncertainty in Measurement	Copy No. Page 1 of 24 Document No. GD07 /04 Revision no. 0 Effective Date. 2020- 05-17

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1. Purpose

Recently, words like quality, calibration, uncertainty in measurement and traceability of measurement of national and international standards, an important aspect of 'Measurement Science', have gained tremendous importance due to the demand and awareness of ISO 9001 and ISO/IEC 17025: 2017 certifications.

The concept of uncertainty as a quantifiable attribute is quite new in the field of measurement science or metrology. The uncertainty of measurement is the parameter associated with the results of a measurement, which characterises the dispersion of the values that could reasonable be attributed to the measurand. The ideal method for evaluating and expressing the uncertainty of the results of a measurement should be universal, internally consistent and transferable.

In the era of global trade, it has become very important that there should be international consensus in the evaluation and expression of the uncertainty of measurement just like universal use of the 'International System of Units' (SI).

The international documents given in 'Normative Reference will help to achieve consensus at the international level to have a common document and full information on how uncertainty statements are arrived at and will also provide a platform for the international comparison of measurement results.

2. Scope:

The scope of this document is to be a guidance document while estimating measurement uncertainty in a variety of laboratories such as calibration and testing laboratories. Measurements which can be treated as outputs of several correlated inputs have been excluded from the scope of this document. This guide document shall address:

(i) Definition of few related terms used in uncertainty in measurement

(ii) Type A and Type B methods

(iii) The definition and the evaluation process
(iv) Combined uncertainty
(v) Calculation of effective degree of freedom and coverage factor
(vi) Expanded uncertainty
(vii) Uncertainty budget
(viii) Reporting of the results
(ix) Proficiency Testing and general statistical methods used for interpretation
(x) Few examples of various parameters under scope of accreditation

3. Normative References:

Based on the following documents, the present guideline for uncertainty in measurement was made:

1. ISO/IEC Guide 98-3:2008 - Uncertainty of measurement - Part 3: Guide to the expression of uncertainty in measurement (GUM: 1995).
2. Evaluation of measurement data - Guide to the expression of uncertainty in measurement, 'JCGM 100:2008- (ISO/IEC 98-3)'.
3. ISO/IEC Guide 99:2007, International vocabulary of metrology - Basic and general concepts and associated terms (VIM)
4. ISO 3534-1 Statistics - Vocabulary and symbols - Part 1: General statistical terms and terms used in probability
5. APLAC TC005-Interpretation and guidance on estimation of uncertainty of measurement in testing
6. IEC GUIDE 115:2007 - Application of uncertainty of measurement to conformity assessment activities in the electrotechnical sector
7. UKAS M3003 edition 3 November 2012- The expression of uncertainty and confidence in measurement
8. EURACHEM / CITAC Guide quantifying uncertainty in analytical measurements (edition 3 - 2012)

9. ISO 21748:2010 Guidance for the use of repeatability, reproducibility and trueness estimates in measurement uncertainty estimation
10. ISO / IEC 17025:2017, General requirements for the competence of testing and calibration laboratories
11. ISO Guide 35, Certification of Reference Materials - General and Statistical Principles, Second Edition 1989

4. Definition

These few terms of general interest have been taken from the —International Vocabulary of Basic and General terms in Metrology.

1. **Measurement**: Process of experimentally obtaining one or more quantity values that can reasonably be attributed to a quantity.
2. **Measurand:** Quantity intended to be measured.
3. **Metrology:** Science of measurement and its applications.
4. **Uncertainty in measurement**: The parameter associated with the result of a measurement that characterizes the dispersion of the values that could reasonably be attributed to the measurement.
5. **Measurement Accuracy / Accuracy of Measurement / Accuracy**: Closeness of agreement between a measured quantity value and a true quantity value of a measurand
6. **Measurement Error / Error of Measurement / Error**: Measured quantity value minus a reference quantity value
7. **Calibration:** Set of operation that establish, under specified conditions, the relationship between values of quantities indicated by a measuring system, or values represented by a material measure or a reference material, and the corresponding values realized by standards.
8. **Metrological Traceability:** Property of a measurement result whereby the result can be related to a reference through a documented unbroken chain of calibrations, each contributing to the measurement uncertainty
9. **Repeatability:** The closeness of agreement between results of successive measurements of the same value of a quantity carried out under repeatability conditions.
    - Repeatability condition**:** Conditions where independent test results are obtained with the same method on identical test items in the same laboratory by the same operator using the same equipment within short interval of time.
10. **Reproducibility:** The closeness of agreement between the results of measurements of the same measurand carried out under reproducibility conditions.

- Reproducibility conditions: It includes a valid statement of conditions changed. The changed conditions may include the principle of measurement, method of measurement, instruments, observer, time, location etc.

11. **Variance:** The expectation of the quadratic deviation of a random variable about its expectation.
12. Thus, the variance of random variable, **z**, with probability density function, **p(z)**, is given by

$$\sigma^2(z) = \int (z - \mu_z)^2 \, p(z) \, dz$$

where μ_z is the expectation of **z**. The variance $\sigma^2(z)$ may be estimated by

$s^2(z_i) = \{1/(n-1)\} \sum (z_i - z)^2$ where $z = 1/n \sum z_i$

z_i are **an** independent observation of z. Where **s(z)** is termed the experimental standard

13. **Standard deviation**: It is the positive square root of the variance. For variance σ^2 equal to v(x) than standard deviation is given as $\sigma = \sqrt{v/(x)}$
14. **Degree of freedom**: In general number of terms in a sum minus number of constraints on the terms of the sum.
15. **Coverage factor**: A number factor used as a multiplier of the standard uncertainty of measurement in order to obtain an expanded uncertainty of the measurement.
16. **Sensitivity coefficient:** The differential change in the output estimate generated by the differential change in that input estimate.

5. Expression and Evaluation of Uncertainty in Measurement:

First of all, model the measurement (mathematical formulation) to be undertaken and then evaluate uncertainty in measurement associated with the input estimates using either Type A or Type B method of evaluation. Finally express the result. In detail these steps are as follows:

5.1. Modelling the Measurement:

The output measurand **Y** is usually determined by **N** input quantities **(X_1, X_2, X_3 ----X_N)** through a function **f** as:

$Y = f(X_1, X_2, X_3 \text{--------} X_N)$(2.1)

The function **f** includes every quantity that contributes a major component of uncertainty to the measurement results. An estimate of the measurand **Y**, denoted by **y**, is obtained from equation **(2.1)** using input estimates **x_1, x_2, x_3 --------x_N**, for the values of **N** input quantities **X_1, X_2, X_3 --------X_N**. The output estimate **y**, which is the result of the measurement, is thus given by:

$y = f(x_1, x_2, x_3 \text{--------} x_N)$(2.2)

The estimated standard deviation of the output estimate **y** is obtained by appropriately combining the estimated standard deviation (termed as standard uncertainty) and denoted by **$u(x_i)$** of each input estimate **x_i**. Each **$u(x_i)$** is evaluated either from Type A or Type B evaluation described in the next section.

5.2. The evaluation of uncertainty in measurement associated with the input estimate x_i.

This is evaluated either by ‘Type A’ or ‘Type B’ method of evaluation. Let us discuss them one by one.

Type A evaluation of standard uncertainty:

In this method of evaluation, the uncertainty is calculated by the statistical analysis of a series of observations. The standard uncertainty is the experimental standard deviation of the mean that follows from an averaging procedure or an appropriate analysis.

a. **Mathematical representation:**

Under the same condition of measurements, if **n** independent observations have been made for one of the input quantities $\mathbf{X_i}$, assume that the repeatedly measured input quantities $\mathbf{X_i}$ are the quantity **Q**. For **n (n>1)** independent observations, the estimate of the quantity **Q** is $\bar{q}$, the arithmetic mean is given as:

$$\bar{q} = (1/n)\,\Sigma q_j \qquad (2.3)$$

The experimental variance $s^2(q)$ is given as

$$s2\,(q) = \{1/(n-1)\}\,\Sigma(qj - \bar{q})2 \qquad (2.4)$$

The best estimate of the variance of the arithmetic mean $\bar{q}$ is the experimental variance of the mean given by;

$$s^2(\bar{q}) = \{s^2(q)/n\} \quad (2.5)$$

The standard uncertainty $u(\bar{q})$ associated with the input estimate $\bar{q}$ is the experimental standard deviation of the mean given as;

$$u(\bar{q}) = s(\bar{q}) \quad (2.6)$$

The graphical illustration of the probability distribution of $\mathbf{x_i}$ that is sampled by means of repeated observations is as shown in **Fig. 1**.

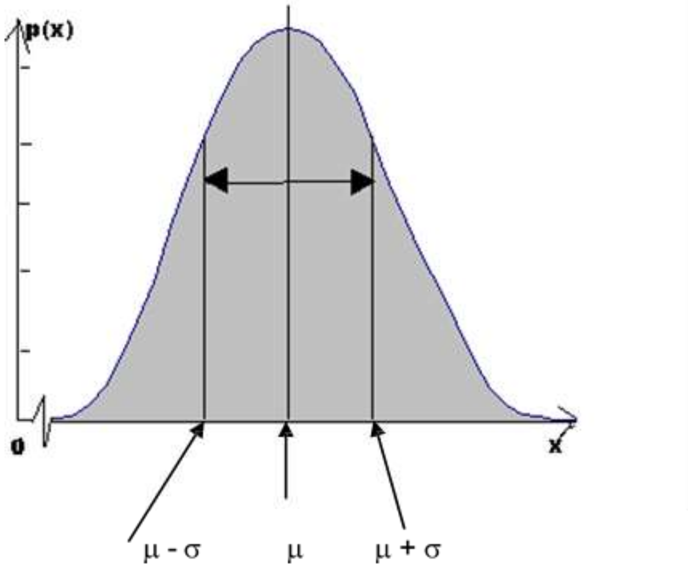

Fig 1: Normal or Gaussian distribution

Where □ is the mean and □ is the standard deviation?

Type B evaluation of standard uncertainty:

In this method of evaluation, the uncertainty is calculated by means other than the statistical analysis of a series of observations. The standard uncertainty is evaluated by the proper use of the available information and calls for insight based on experience and general knowledge. Values may be derived from

- Previous measurements data
- Experience with or general knowledge of the behaviour and properties of relevance materials and instruments
- Manufacturer's specifications
- Data provided in calibration and other certificates
- Uncertainties assigned to reference data taken from handbooks

a. Mathematical representation:

In the calculation of Type B uncertainty, insight based on experience and general awareness is most important. It may be said that it is a specialised skill that can be learned with constant efforts. The most common situations, which appear in measurements, may be treated as follows:

A. Single value is known for the quantity, X_i:

In such cases, a single measured value, a resultant value of a previous measurement, a reference value from the literature, or a correction value will be used for $\mathbf{x_i}$. The standard uncertainty $\mathbf{u(x_i)}$ associated with $\mathbf{x_i}$ is to be adopted where it is given. If data of this kind are not available, the uncertainty has to be evaluated on the basis of experience.

for examples:

i. A calibration certificate states that the mass of a given body of 10 kg is 10.000 650 kg. The uncertainty at 3σ is given as 300 mg.

   In such case, the standard uncertainty is then simply, u(m) = 300/3 = 100 mg and estimated variance is $u^2(m) = 1 \times 10^{-2}\ g^2$

ii. Suppose, in the above example the quoted uncertainty defines an interval having a 95% level of confidence.

   In such case, the standard uncertainty is then simply, u(m) = 300/2= 150 mg where we have taken 2.0 as the factor corresponding to the above level of confidence, assuming ‘normal distribution’, unless otherwise stated.

ii. A calibration certificate states that the mass of a given body of 10 kg is 10.000 650 kg. The uncertainty at 3σ is given as 300 mg.

   In such case, the standard uncertainty is then simply, u(m) = 300/3 = 100 mg and estimated variance is $u^2(m) = 1 \times 10^{-2}\ g^2$

iii. Suppose, in the above example the quoted uncertainty defines an interval having a 90% level of confidence.

In such case, the standard uncertainty is then simply, u(m) = 300/1.64 = 182.9 mg where we have taken 1.64 as the factor corresponding to the above level of confidence, assuming ‘normal distribution’, unless otherwise stated.

B. Probability distribution assumed for the quantity, X_i :

In such cases, the appropriate expectation or expected value and the square root of the variance of this distribution have to be taken as the estimate $\mathbf{x_i}$ and the associated standard uncertainty $\mathbf{u\ (x_i)}$, respectively.

C. Upper and lower limits a_+ and a_- can be estimated for the quantity, X_i :

I. Trapezoidal distribution:

One may note, in many realistic cases it is more likely to have values near the midpoint than those near the bound. Thus, we may assume a more general distribution for **Xi** i.e. trapezoidal distribution having equal sloping sides (an isosceles trapezoid) a base of width $\mathbf{(a_+ - a_-) = 2a}$, and a top of width $\mathbf{2\beta a}$, where $\mathbf{0} \le \beta \le \mathbf{1}$. The expectation of **Xi** is **xi** given as

$$x_i = (a_+ + a_-)/2 \quad (2.7)$$

and its associated variance is

$$u^2(x_i) = a^2(1+ \beta^2)/6 \quad (2.8)$$

Depending on the values of β, two cases arise

Case I: for $\beta \to$ **1**, it is rectangular distribution

$$u^2(x_i) = a^2/3 \quad (2.9)$$

Since, this is normally used distribution further details are given in part (ii).

Fig. 2 gives the graphical representation of this distribution.

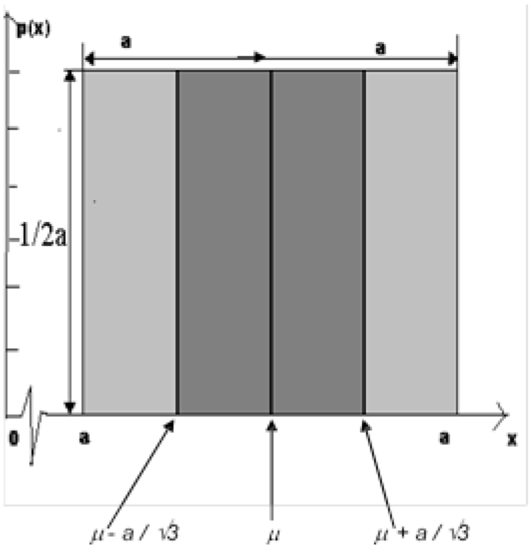

Fig 2 : Schematic of the rectangular distribution

Case II: for β **= 0**, it is triangular distribution

$$u^2(x_i) = a^2/6 \quad (2.10)$$

Fig. 3 gives the graphical representation of this distribution.

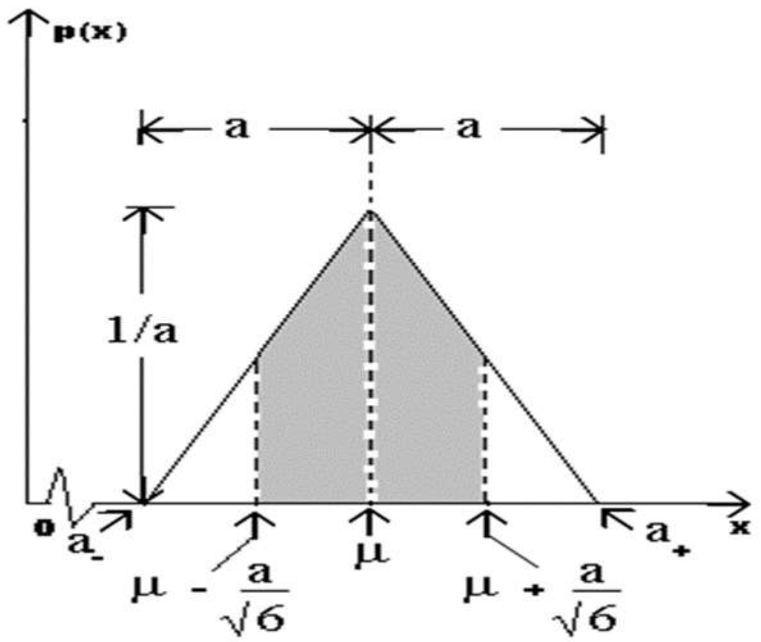

Fig 3: Schematic of triangular distribution

Please note: *The trapezoidal distribution is equivalent to the convolution of two rectangular distributions.*

II. Rectangular distribution:

In such case, we do not have a specific knowledge about the possible values of Xi within its estimated bounds $\mathbf{a_-}$ and $\mathbf{a_+}$, one could only assume that it was equally probable for $\mathbf{X_i}$ to take any value within these bounds, with zero probability of being outside them. A uniform or rectangular distribution has to be assumed for the possible variability of the input quantity X_i. Then x_i , is the midpoint of the interval,

$x_i = (a_+ + a_-)//2$(2.11)

with the associated variance as

$u^2(x_i) = (a_+ - a_-)^2/12$(2.12)

if difference between the bounds is denoted by **2a**, then equation (2.12) may be written as

$u^2(x_i) = a^2/3$(2.13)

Fig. 2 gives the graphical representation of this distribution.

For example:

Let us take the case of mass measurements dealt earlier, if the calibration certificate states, “the error in this value should not exceed 0.4 g”. Based on this limited information, it is reasonable to assume that the value of the mass lies between 10.000250 kg and 10.001050 kg and one can use the above equations to obtain the standard uncertainties etc.

III. Triangular distribution:

When it is known that most of the values are likely to be near the centre of the distribution, then one must use the triangular distribution. The standard uncertainty is computed as given equation (2.14).

$u^2(x_i) = a^2/6$(2.14)

Fig. 3 gives the graphical representation of this distribution

D. U-shape probability distribution:

This U-shape probability distribution is used in the case of mismatch uncertainty in ratio and microwave power frequency measurements. The standard uncertainty is computed as:

$u^2(x_i) = (2\Gamma_S \Gamma_L)^2/2$(2.15)

where, Γ_S and Γ_L are the reflection coefficients of the source and load respectively.

Fig. 4 gives the graphical representation of this distribution.

Please note: *In many practical measurement situations where the bounds are asymmetric, it may be appropriate to apply a correction to the estimate and calculate the new symmetrical bounds.*

2.3 Combined Standard Uncertainty:

Combined standard uncertainty may be of two types

I. Uncorrelated input quantities

II. Correlated input quantities

I. Uncorrelated input quantities:

In this case all the input quantities are **independent** and the combined standard uncertainty $\mathbf{u_c(y)}$ is the positive square root of the combined variance $\mathbf{u_c^2(y)}$ which is given by:

$$\mathbf{u_c^2(y) = \Sigma\ [(\partial f\ /\ \partial x_i)^2\ u^2(x_i)] = \Sigma[(c_i)^2\ u^2(x_i)]} \quad \mathbf{(2.16)}$$

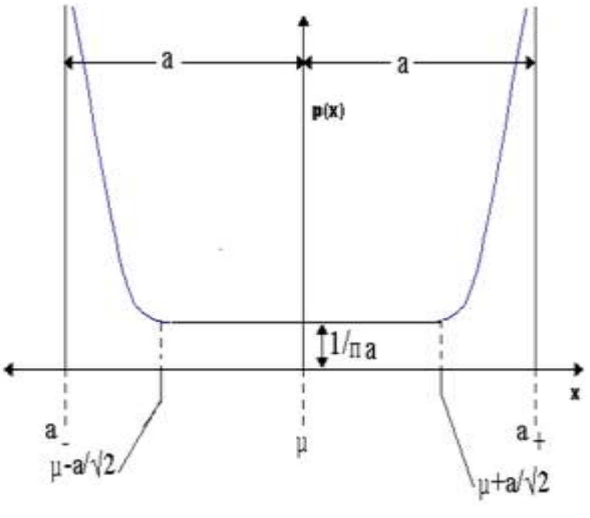

Fig 4 : Schematic of U-shaped distribution

Where, $\mathbf{c_i}$ is known as sensitivity coefficient and describes how much extent the output estimate **y** is influenced by variation of the input estimate $\mathbf{x_i}$**;** and function **f** is given by equations (2.1) and (2.2) respectively. Let us consider the two simple cases:

i. If function **f** is a sum or difference of the input quantities $\mathbf{X_i}$**,**

$$f\,(X_1, X_2, X_3, \text{--------}\ X_n) = \Sigma\, p_i\, X_i \quad (2.17\ a)$$

The output estimates as per equation (2.2) is given by the corresponding sum or difference of the inputs estimates

$$y = \Sigma\, p_i\, x_i \quad (2.17\ b)$$

Where the sensitivity coefficients equal $\mathbf{p_i}$ and the combined uncertainty is given by

$$u_c^2(y) = \Sigma((p_i)^2\, u^2(x_i) \quad (2.17\ c)$$

Each $\mathbf{u(x_i)}$ is a standard uncertainty evaluated as Type A or Type B.

ii. If function **f** is a product or quotient of the input quantities $\mathbf{X_i}$**,**

$$f\,(X_1, X_2, X_3, \text{--------}\ X_n) = \Pi\, X_i^{\ pi} \quad (2.17\ d)$$

the output estimates as per equation (2.2) is given by the corresponding product or quotient of the inputs estimates

$$y = \Pi\, x_i^{\ pi} \quad (2.17\ e)$$

The sensitivity coefficients equal $\mathbf{p_i\ y/x_i}$ and the relative combined uncertainty is given by

$$[u_c(y)/\ y]^2 = \Sigma[p_i\, u(x_i)/\ x_i\ ]^2 \quad (2.17\ f)$$

The combined standard uncertainty $\mathbf{u_c(y)}$ is an estimated standard deviation and characterises the dispersion of the values that could reasonably be attributed to the measurand **y**.

II. Correlated input quantities:

In this case the input quantities are correlated, the appropriate expression for the combined variance $\mathbf{u_c^2(y)}$ associated with the result of a measurement is;

$$u_c^2(y) = \Sigma\Sigma\ [(\partial f\ /\ \partial x_i)\ (\partial f\ /\ \partial x_j)\ u(x_i\ , x_j)] \quad (2.18\ a)$$

Where x_i and x_j are the estimates of x_i and x_j and $u(x_i, xj) = u(x_j, x_i)$ is the estimated covariance associated with x_i and x_j, the degree of correlation between them is characterised by the estimated correlation coefficient given as;

$$r(x_i, x_j) = \frac{u(x_i, x_j)}{u(x_i)u(x_j)} \quad (2.18\ b)$$

2.4 Calculation of the Standard Uncertainty of the Output estimate:

Consider the simple case of uncorrelated input quantities; the standard uncertainty of the output estimate **y** is given by;

$$u^2(y) = \Sigma\, u_i^2\,(y) \quad (2.19\ a)$$

$u_i\,(y)$ is the contributions to the standard uncertainty associated with the x_i given as

$$u_i(y) = c_i\, u(x_i) \quad (2.19\ b)$$

Where, c_i is **known as sensitivity coefficient** and describes how much extent the output estimate **y** is influenced by variation of the input estimate x_i .

The sensitivity coefficient is calculated as follow:

$$c_i = \partial f / \partial x_i \quad (2.20)$$

Combined standard uncertainty is given by:

$$u_c^2(y) = \Sigma\, c_i^2\, u^2(x_i) = \Sigma\, [c_i\, u(x_i)]^2 \quad (2.21)$$

2.5 Expanded uncertainty:

The additional measure of uncertainty that meets the requirements of providing an interval is termed expanded uncertainty and is denoted by **U**.

The expanded uncertainty **U** is obtained by multiplying the combined standard uncertainty $u_c(y)$ by a coverage factor **k**.

$$U = k\, u_c(y) \quad (2.22)$$

2.6 Result of measurement:

The result of a measurement is then conveniently expressed as

Y = y ± U(2.23)

which is interpreted to mean that the best estimate of the value attributable to the measurand **Y** is **y** and that **y-U** to **y+U** is an interval that may be expected to encompass a large fraction of the distribution of values that could reasonably be attributed to **Y**. Such an interval is also expressed as

y-U ≤ Y ≤ y+U(2.24)

2.7 Reporting of result:

It should have the following content:

"The reported expanded uncertainty in measurement is stated as the standard uncertainty in measurement multiplied by the coverage factor k which for a t-distribution with, ν_{eff}, effective degrees of freedom corresponds to a coverage probability of approximately 95%".

2.8 Uncertainty budget:

Uncertainty budget includes a list of all sources of uncertainty together with the associated standard uncertainties of measurement and methods of evaluating them. For the sake of clarity, it is better to present the data relevant to this analysis in the form of a table. Table 1 gives an example of the uncertainty budget and data.

Table 1. Uncertainty budget;

Quantity X_i	Estimated value x_i	Limits $\pm(x_i)$	Uncertainty (μV)	Probability Distribution/Type A or B	Sensitivity Coefficient c_i	Degree of freedom ν_i	Uncertainty Contribution $u_i(y)$ [μV]	$\{u_i^2(y)\}$ $(\mu V)^2$
V_s	1.0179285 V		0.5 μV	normal type B	1	90	0.5	0.25
δV_D	-0.6 μV	0.1 μV	0.058 μV	rectangular type B	1	∞	0.058	33.64×10^{-4}
δV_I	-190.632 μV		0.035 μV	normal type A	1	9	0.035	12.25×10^{-4}
δV_2	0 μV	0.1 μV	0.058 μV	rectangular type B	1	∞	0.058	33.64×10^{-4}
δt_S	0 K	2.0 mK	1.155 mK	rectangular type B	0.104 μV/mK	∞	0.120	1.44×10^{-2}
δt_X	0 K	1.0 mK	0.577 mK	rectangular type B	0.104 μV/mK	∞	0.060	3.6×10^{-3}
δE	0 μV	0.1 μV	0.058 μV	rectangular type B	1	∞	0.058	33.64×10^{-4}

V_x	1.0181185 V							$\sum$ = 28.60x10^{-2} $\sqrt{\sum}$ = 0.535 (μV)

Reported result:

The measured average voltage of the unknown cell is 1.0181185 V ±1.07 μV

The reported expanded uncertainty of measurement is stated as the standard uncertainty of measurement multiplied by the coverage factor k = 2, which for a normal distribution corresponds to a coverage probability of approximately 95.45 %.

6. Effective Degree of Freedom and Coverage Factor:

If $u_c^2(y)$ is the sum of two or more estimated variance components $u_i^2(y)$, then Student's **'t'** -distribution in general will not describe the distribution. However, it may be approximated by a Student's **'t'** - distribution with an effective degree of freedom, ν_{eff}, obtained by Welch-Satterthwaite formula given as:

$$\nu_{eff} = \frac{\{u_c^4(y)\}}{\sum_{i=1}^{N} \{u_i^4(y)\}/\nu_i} \qquad (4.1)$$

Where the $u_i(y)$ **[i = 1, 2, ………… N]** is the contribution to the standard uncertainty associated with the output estimate **y** resulting from the standard uncertainty associated with the input estimate X_i. ν_i is the degree of freedom for each standard uncertainty component.

After evaluating ν_{eff} if we find it is not an integer, interpolate or truncate to the next lower integer and the corresponding coverage factor **k** value is obtained from the table of Student's **'t'**- distribution.

The calculation of the degree of freedom ν_i for the two types of the evaluation may be as follow:

For Type A, when **n** numbers of observations are taken then

$$\nu_i = n-1 \qquad (4.2)$$

For Type B, when estimated upper and lower limits are known, then

$$\nu_i \rightarrow \propto \qquad (4.3)$$

- If the limits themselves have some uncertainty, then a lesser number of degrees of freedom must be assigned. The ISO guide to the expression of uncertainty in measurement (GUM) gives a formula that is applicable to all distributions. It is given as:

$$\nu \approx \tfrac{1}{2} [\Delta u (x_i) / u (x_i)]^{-} (4.4)$$

Where:

$\Delta u (x_i) / u (x_i)$ is the relative uncertainty in the uncertainty.

For example, consider the following cases along with the degree of freedom.

$\Delta u (x_i) / u (x_i)$ (percent)	**Degree of freedom**
10	**50**
25	**8**
50	**2**

From the above table following points may be noted:

- It is better to try to determine the limits more definitely, particularly if the uncertainty is a major one.
- It is the interest to note that equation (4.4) tells us that when we have made 51 measurements and taken the mean, the relative uncertainty in the measurement of the mean is 10%. This shows that even when many measurements are taken, the reliability of the uncertainty is not necessary any better than when a Type B assessment is made.
- It is usually better to relay on prior knowledge rather than using an uncertainty based on two or three measurements.

7. Proficiency Testing and General Statistical Methods used for Interpretation:

Proficiency Testing (PT) is one of the quality assurance tools to enable a laboratory to compare their performance with similar laboratories. It helps them to understand/demonstrate their competence to an accreditation body or third party, to take any necessary remedial action and to facilitate improvement.

For conducting the PT, one should comply with ISO/IEC 17043:2010.

For interpretation of the data the following statistical methods are normally used.

Let,

$\mathbf{x_i}$ be the participants results and

X the assigned or reference value.

Then **Z** score and $\mathbf{E_n}$ number may be evaluated as follow:

(a) $\mathbf{E_n}$ or normalized error:

$$E_n = \frac{x_i - X}{\sqrt{[u^2(x_i) + u^2(X)]}} \qquad \begin{array}{ll} \le 1 & \text{(satisfactory)} \\ > 1 & \text{(un-satisfactory)} \end{array}$$

(b) **Z** score:

$$Z = \frac{x_i - X}{S} \qquad \begin{array}{ll} \le 2 & \text{(satisfactory)} \\ 2 < |Z| \le 3 & \text{(questionable)} \\ |Z| > 3 & \text{(un-satisfactory)} \end{array}$$

Where **S** is estimate/measure of variability/standard deviation

Annex I

Table 1: Student t-distribution for degrees of freedom v.

The t-distribution for v v defines an interval $-t_p(v)$ to $+t_p(v)$ that encompasses the fraction p of the distribution. For p = 68.27%, 95.45%, and 99.73%, k is 1, 2, and 3, respectively.

Degrees Freedom	**Fraction p in percent**					
(v)	**68.27**	**90**	**95**	**95.45**	**99**	**99.73**
1	1.84	6.31	12.71	13.97	63.66	235.80
2	1.32	2.92	4.30	4.53	9.92	19.21
3	1.20	2.35	3.18	3.31	5.84	9.22
4	1.14	2.13	2.78	2.87	4.60	6.62
5	1.11	2.02	2.57	2.65	4.03	5.51
6	1.09	1.94	2.45	2.52	3.71	4.90
7	1.08	1.89	2.36	2.43	3.50	4.53
8	1.07	1.86	2.31	2.37	3.36	4.28
9	1.06	1.83	2.26	2.32	3.25	4.09
10	1.05	1.81	2.23	2.28	3.17	3.96
11	1.05	1.80	2.20	2.25	3.11	3.85
12	1.04	1.78	2.18	2.23	3.05	3.76
13	1.04	1.77	2.16	2.21	3.01	3.69
14	1.04	1.76	2.14	2.20	2.98	3.64
15	1.03	1.75	2.13	2.18	2.95	3.59
16	1.03	1.75	2.12	2.17	2.92	3.54
17	1.03	1.74	2.11	2.16	2.90	3.51
18	1.03	1.73	2.10	2.15	2.88	3.48
19	1.03	1.73	2.09	2.14	2.86	3.45
20	1.03	1.72	2.09	2.13	2.85	3.42
25	1.02	1.71	2.06	2.11	2.79	3.33
30	1.02	1.70	2.04	2.09	2.75	3.27
31	1.000	1.645	1.960	2.000	2.576	3.000

Revision No.	Date approved	Revision History